Design of a Low Loss Metallo-Dielectric EBG Waveguide at Submillimeter Wavelengths

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Abstract—In this letter we show the viability of using concentric cylindrically-periodic (CP) dielectrics to realize low-loss propagation at submillimeter wavelengths. Most of the power in the CP Waveguide (WG) is confined to an air core propagating a TE_{01} mode. The TM_{11} mode degeneracy is removed over a very wide bandwidth by a series of periodically spaced dielectric rings that are optimized in thickness and spacing. The new waveguide differs from classical Bragg fibers in that we use only a small number of dielectric layers (2–4), and terminate the outer layer with an external metal coating. We include several designs and describe the optimization procedure that was used to realize structures with low propagation and bend loss.

Index Terms—Bragg fibers, electromagnetic bandgap (EBG), optimization, THz waveguide.

I. INTRODUCTION

SUBMILLIMETER-WAVE (THz) instruments and applications would benefit greatly from the availability of flexible, low-loss waveguide. However, strong inherent dielectric absorption in non-conducting solids and high skin-depth losses in conductors have made it extremely difficult to realize either traditional metallic guides or any of the wide variety of available dielectric guides without incurring large propagation losses, poor mode confinement and uniformity, or both. To overcome the wall losses, both metal and dielectric WGs have been implemented with oversized hollow cores [1], [2]. However, this implies that many modes are above the cut-off and they are readily excited at bends or small discontinuities. Similarly, to avoid the high dielectric losses, guides structured such that most of the propagating field lies outside the dielectric region (ribbon guides [3]) have been constructed, but here the mode confinement is not very strong and there is significant mode conversion around bends.

CPWGs have demonstrated very low loss in the infrared (IR) and optical bands [4], [5]. However, the successful Bragg fibers have been fabricated using a large number of layers and/or very large core dimensions. In this letter we derive and optimize an alternative CPWG with a small number of layers and core dimension that is terminated by a metal wall. This wall adds a modest conductor loss contribution but eliminates the radiation
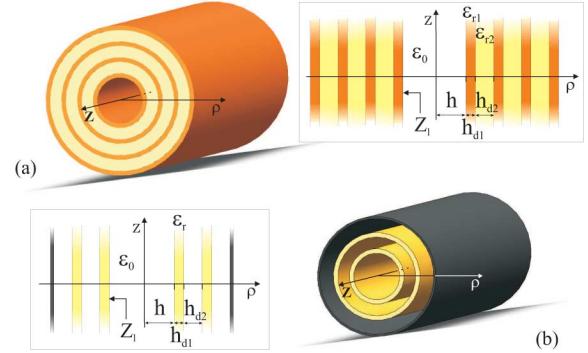
Fig. 1. Geometry of a CPWG: (a) open and (b) new closed conductor geometries. Insets show the equivalent 2-D structures.

loss that would otherwise dominate. It has been shown that the modes in the core of the CPWG resemble those of a circular WG (CWG) [6]. The proposed CPWG (Fig. 1) works on the lowest loss TE_{01} mode. It is well known that in bends or discontinuities the TE_{01} couples to the lossy TM_{11} mode since these two modes are degenerate. To reduce this mode coupling, an optimization technique has been developed that breaks the degeneracy and maintains the low loss TE_{01} mode around bends. It was found that a CPWG structure of 4.1 λ_0 diameter composed of only two properly dimensioned and positioned fused quartz rings would have a theoretical propagation loss of 2.5 dB/m and could tolerate a bend radius of $< 15\ \lambda_0$ without incurring undo mode conversion.

II. ESTABLISHED AND NEW GEOMETRIES

Bragg fibers have shown extremely good propagation characteristics in the optical and IR bands. They were first introduced by [7] and fabricated by [4], [5]. They consist of cylindrically periodic layers surrounding an air core [Fig. 1(a)]. The periodicity is realized by employing two different dielectrics, one with high, and the other with low permittivity. These structures propagate a leaky wave mode with a complex propagation constant $k_z = \beta - j\alpha$ whose radiation is minimized by using a large number of periods (typically 15 or more) and a large core dimension. The optimal dielectric thicknesses realize the same "resonant condition" of planar leaky wave antennas [8]

$$\frac{h}{\lambda_0} = \frac{0.5}{\cos(\theta)}, \quad \frac{h_{di}}{\lambda_0} = \frac{0.25}{\sqrt{\epsilon_{ri} - \sin^2(\theta)}} \tag{1}$$

where $\sin(\theta) = \beta/k_0$ and $i = 1, 2$.

Classical Bragg fibers at optical wavelengths generally employ a large diameter core ($\varnothing \approx 20\ \lambda$) to reduce the propagation loss. The fundamental mode is usually designed to propagate at

the speed of light ($\theta \approx 90°$). In order to have strong field confinement ϵ_{r2} must be significantly larger than ϵ_{r1}. Moreover, at optical frequencies the bends are usually very large in terms of the wavelength. Thus they introduce negligible cross-coupling between modes and, therefore, low losses. As a matter of the fact for large θ the TM mode will tend to radiate due to the presence of the Brewster angle.

Although the optical CPWG works quite well, such a waveguide cannot be easily transposed to submillimeter wavelengths for several reasons. First, a direct scaling would result in large waveguide ($\oslash \approx 8$ mm at 750 GHz), making it potentially very inflexible. Second, there exist no dielectric pairs with the low loss and high permittivity ratios needed to realize efficient guide structures with a reasonable number of periods. Finally, there is no demonstrated fabrication technique that can be used to form the types of geometries that are realized with typical fiber draw processes. Here we study the possibility of using the Bragg concept but with a single dielectric having a low permittivity and a small number of periods consistent with available low loss submillimeter wave materials such as quartz, Teflon, polyethylene and silicon.

Bragg fibers can be analyzed by looking at the leaky wave poles as described in [7]. In particular the procedure in [8] has been used to derive both the propagation constants and the field distributions. The propagation constant for fused quartz ($\epsilon_r = 4$) is shown in Fig. 2(a). The open structure is designed using (1). The radiation loss for three periods has been calculated to be 65 dB/m at 750 GHz.

Using a transmission line analogy, the "resonant condition" transforms a high impedance at the outer wall to a low impedance (Z_l) at the core by stacking several quarter wavelength steps [see Fig. 1(a)]. This geometry thus provides a null in the tangential electric field at the start of each period, as can be seen in Fig. 2(b). If a metallic boundary is placed at any of these points, where the impedance is low, one would expect the mode properties not to be significantly altered. Fig. 2(a) also shows the propagation constant when two periods are closed by an outer metal boundary as in Fig. 1(b).

The degeneracy between the TM_{11} and TE_{01} of a CWG makes the coupling between the two modes along bends or discontinuities independent of the waveguide radius [9]. In both the open and closed CPWG structures, the EH_{11} mode presents a propagation constant β very similar to the TE_{01} mode [see Fig. 2(a)]. In the open structure, the attenuation constant of the EH_{11} mode is very large and therefore the coupling between the two modes at the end of a bend is small. In the closed structure however, the propagation constant of both modes lie close to the real axis and therefore the coupling is strong. In order to reduce the EH_{11} and TE_{01} mode coupling and then the losses introduced by the bend, a design that breaks their degeneracy, but without increasing the propagation loss, is required.

III. OPTIMIZATION PROCEDURE

The CPWG structure is optimized by finding the minimum of a constrained nonlinear multivariable function. Matlab™ provides such a routine: *fmincon*. We perform the optimization for a single frequency point (750 GHz). The function to be minimized is the sum of the WG's dielectric and conductor losses. In particular, these losses have been calculated by using conventional
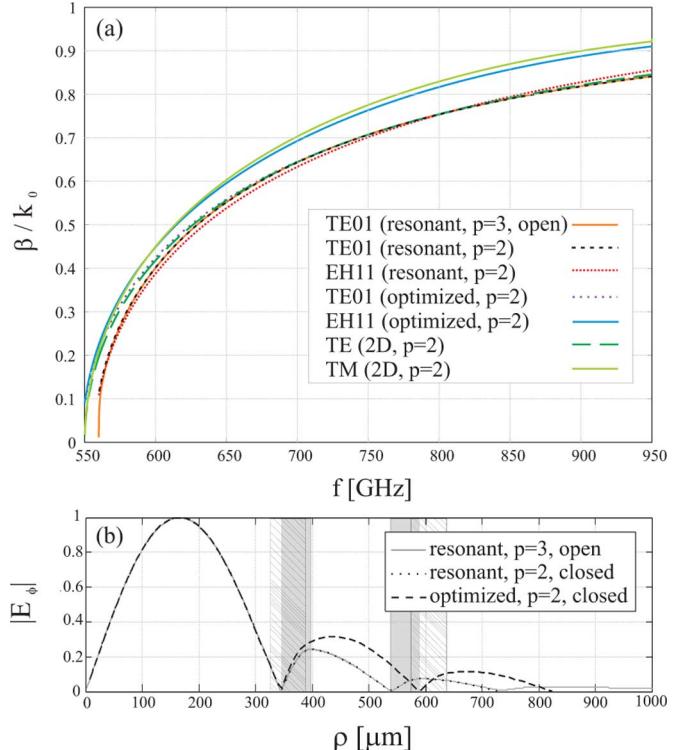
Fig. 2. (a) Propagation constants (p is the number of periods) for: resonant CPWG $h = 346$ μm, $h_{d1} = 51$ μm, $h_{d2} = 141$ μm; optimized CPWG ($R_{\text{opt}} = 10\,\lambda_0$), $h = 326$ μm, $h_{d1} = 62$ μm, $h_{d2} = 187$ μm; and 2-D WG, $h = 262$ μm, $h_{d1} = 62$ μm, $h_{d2} = 187$ μm. (b) TE_{01} electric field distribution at 750 GHz. The grey and dash areas indicate the dielectric dimensions of the resonant and optimized CPWG, respectively.

perturbation formulas with $\tan \delta = 2 \cdot 10^{-4}$ and $\sigma = 10^7$ S/m. The minimization is subject to a nonlinear inequality and a nonlinear equality. The inequality is related to the fact that we would like to propagate with bends having a curvature radius smaller than set R_{opt}. For a CWG this radius can be related to the difference between the propagation constants $\Delta\beta = \beta_{TE_{01}} - \beta_{EH_{11}}$ as described in [9]: $(4.64h)/(\Delta\beta\lambda_0) < R_{\text{opt}}$ for a ~1 dB loss. The equality relates to a fixed value of the propagation constant $\beta_{TE} = \beta_{TE}^{\text{opt}}$. Here we consider $\beta_{TE}^{\text{opt}}/\beta_0 = 0.7$ as an example case. This assumption introduces some dispersion if pulsed signals are used, but it is compatible with heterodyne, continuous wave and direct detection instruments of typical bandwidth.

In the optimization procedure several approximations are used to simplify the analysis. First, we undertake the dispersion analysis for a 2-D planar equivalent structure instead of the full 3-D cylindrical structure (See Fig. 1). The main reason for using a 2-D approximation is that the propagation constant of the structure can be calculated with an analytical formula as given in [10]

$$\beta_{\rho TE} = \frac{\pi \zeta_0 k_0}{h\zeta_0 k_0 - jZ_l^{TE}} \tag{2}$$

$$\beta_{\rho TM} = \frac{\pi}{2h} + \frac{1}{2}\sqrt{\left(\frac{\pi}{h}\right)^2 + 4j\frac{Z_l^{TM}k_0}{h\zeta_0}} \tag{3}$$

where $Z_l^{TE/TM}$ is the TE/TM impedance at the inner air-dielectric interface dielectric boundary (see Fig. 1), and
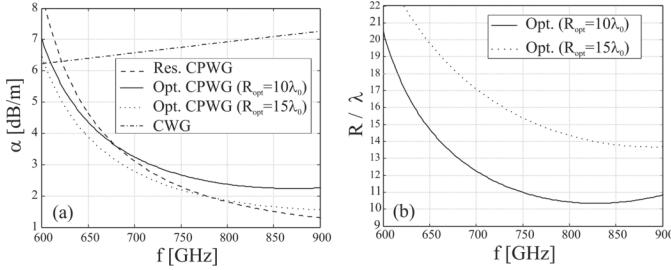
Fig. 3. (a) Propagation loss. The CWG ($\oslash = 1.65$ mm.) case considers only conductor losses. (b) Bend radius associated to 1 dB loss for the resulting $\Delta\beta$, normalized to the wavelength.

Fig. 4. Full wave simulations of a 6 mm bend (optimized CPWG for $R_{\text{opt}} = 10 \lambda_0$): (a) S-parameters and (b) electric field distribution (750 GHz).

$\beta = \sqrt{k_0^2 - \beta_\rho^2}$. During the optimization procedure, the TE propagation constant is constrained to be $\beta_{\text{TE}}^{\text{opt}}$ and, therefore, Z_l^{TE} is calculated assuming $\theta_{\text{TE}} = \sin^{-1}(\beta_{\text{TE}}^{\text{opt}}/k_0)$. The TM propagation constant is then altered, and one needs to track the change of θ_{TM} during the optimization process.

The second approximation concerns the losses. They are calculated by using the fields of the equivalent 2-D structure. The TE_{01} mode is rotationally symmetric, and therefore we introduce only a very small error if we assume that the fields along ρ are those of the 2-D structure. The main difference between the 3-D and 2-D structures is the core dimension h. As a matter of fact, the core of the 3-D CPWG is about 1.22 times larger than that of the 2-D WG, due to the impact of the zero of the Bessel function derivative $J_0'(k_\rho \rho)$. This factor is included in the inequality condition.

The optimization procedure uses (1) as a starting point. The dimensions of the optimized structure, using two periods and with a desired bend radius of $10 \lambda_0$, are given in the caption of Fig. 2. In the same figure, the propagation constants of the 2-D and 3-D structures are shown. It is worth noting that the 2-D TM and EH_{11} propagation constants are slightly different. This is due to the fact that EH_{11} is not rotationally symmetric and consequently its propagation constant depends on both the 2-D TE and TM solutions. Fig. 3(a) shows the propagation loss of the TE_{01} mode in the optimized and resonant conditions. We can see that both losses are comparable. We have also included the loss of the TE_{11} of a hollow CWG with the same outer dimension as the optimized CPWG structure for comparison. Fig. 3 also shows the results of the optimization for a desired bend radius of $15 \lambda_0$. This structure presents lower propagation losses than the previous one and it is slightly smaller ($D = 1.63$ mm).

IV. BEND LOSS

In this section we compare the bend loss behavior of the resonant and optimized CPWG, and of a hollow CWG (TE_{11}). Fig. 3(b) shows the radius of curvature that, accordingly to formulas in [9], provides a bend loss of ~1 dB. To validate our approximations, we simulated the final structure with CST Microwave Studio$^{\text{TM}}$ using a 90° bend. Fig. 4(a) shows the calculated S-parameters and (b) the electric field distribution in the optimized CPWG. A bend loss lower than 1 dB over the entire frequency range (630–870 GHz) has been obtained.

We have also performed simulations using the same bend radius for the resonant CPWG and the hollow CWG structures. For the CWG over 3 dB bend loss is present due to coupling
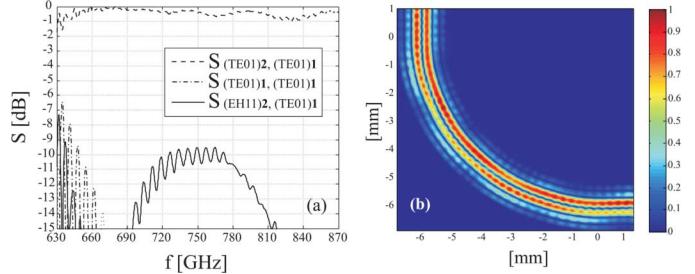
to the higher order modes. While in the case of the resonant CPWG, the power at the end of the bend is all coupled to the EH_{11} mode due to the modes degeneracy.

V. CONCLUSION

In this letter we have presented a new high frequency WG that has both low propagation and low bend losses. The new CPWG has a significantly smaller diameter than an equivalent Bragg fiber construct allowing a greater degree of mechanical flexibility. Using an optimization procedure of the dielectric layer thickness, we can break the degeneracy between the TE_{01} and EH_{11} reducing bend losses. The CPWG has both lower propagating and lower bend losses than a hollow CWG with the same outer dimension.

The next step will be the fabrication of such a WG. A possible method includes using fiber extrusion techniques with thin radial supports. The excitation of the TE_{01} mode could be performed with standard serpentine or quasi-optical mode converters. The optimization procedure can be used to derive design curves depending on the application trade-offs, and has some tuneable parameters that can be changed. These aspects will be clarified in a future longer publication.

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